

AP Calculus (BC) Summer Assignment (60 points)

This packet is a review of some Precalculus topics and some Calculus topics. It is to be done NEATLY and on a SEPARATE sheet of paper. Use your discretion as to whether you should use a calculator or not. When in doubt, think about if I would use one – that should guide you! Points will be awarded only if the correct work is shown, and that work leads to the correct answer. Have a great summer and I am looking forward to seeing you in September. ☺

Parts 1 – 6: Due _____ Tuesday, September 8, 2020 __ (First Day of School) _____

All work needs to be received by the above due dates (emailed with scanned attachments in a word document or pdf)

LATE work will not be accepted!

Part I: First, let's whet your appetite with a little Precalc! (10 points)

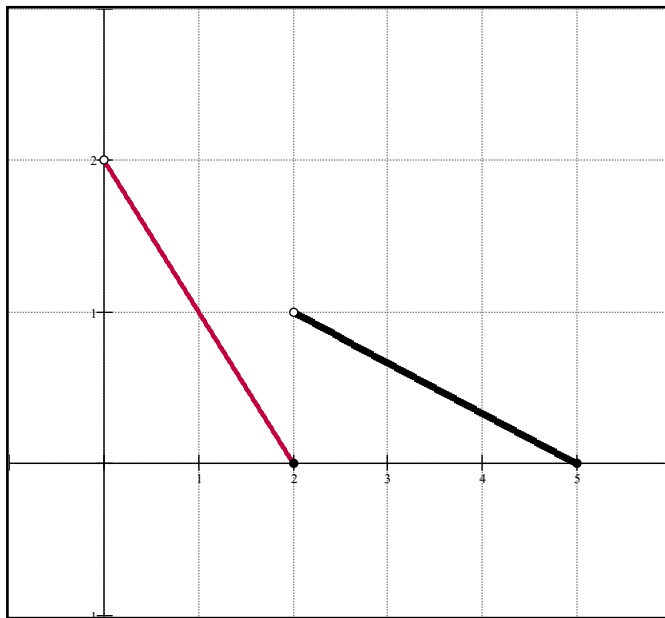
- 2 1) For what value of k are the two lines $2x + ky = 3$ and $x + y = 1$
(a) parallel? (b) perpendicular?

- 2 2) Consider the circle of radius 5 centered at $(0, 0)$. Find an equation of the line tangent to the circle at the point $(3, 4)$ in slope intercept form.

- 2 3) Graph the function shown below. Also indicate any key points and state the domain and range.

$$f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \frac{3}{2}x + \frac{3}{2}, & 1 \leq x \leq 3 \\ x + 3, & x > 3 \end{cases}$$

- 2 4) Write a piecewise formula for the function shown. Include the domain of each piece!



- 2 5) Graph the function $y = 3e^{-x} - 2$ and indicate asymptote(s). State its domain, range, and intercepts.

Part II: Unlimited and Continuous! (10 points)

For #1-2 below, find the limits, if they exist. (#1-8 are 1 pt each)

1) $\lim_{x \rightarrow 4} \frac{2x^3 - 7x^2 - 4x}{x - 4}$

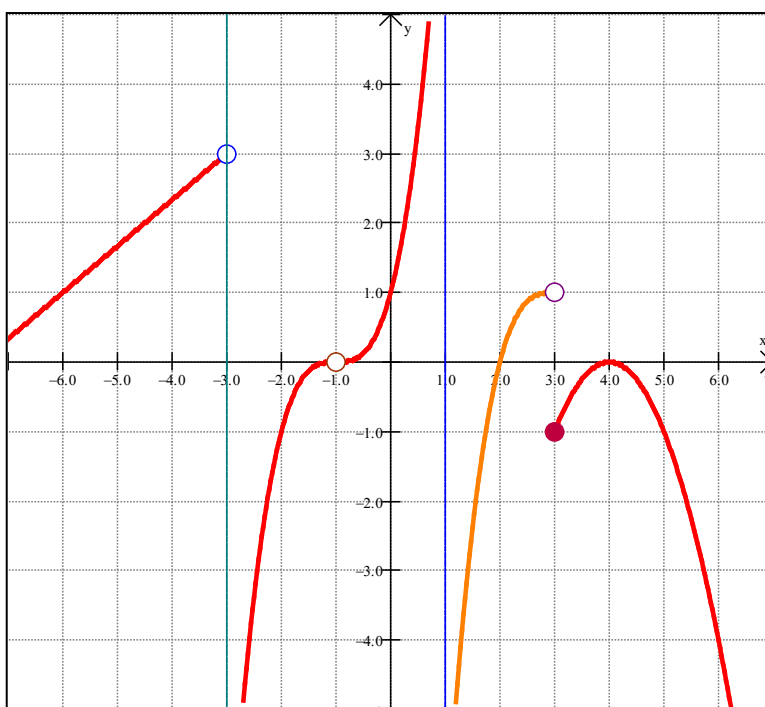
2) $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{9 - x}$

For #3-4, explain why each function is discontinuous and determine if the discontinuity is removable or non-removable.

3) $g(x) = \begin{cases} 2x - 3, & x < 3 \\ -x + 5, & x \geq 3 \end{cases}$

4) $h(x) = \frac{\sqrt{x^2 - 10x + 25}}{x - 5}$

For #5-8, determine if the following limits exist, based on the graph below of $p(x)$. If the limits exist, state their value. Note that $x = -3$ and $x = 1$ are vertical asymptotes.



5) $\lim_{x \rightarrow 1^-} p(x)$

6) $\lim_{x \rightarrow -3^-} p(x)$

7) $\lim_{x \rightarrow 3} p(x)$

8) $\lim_{x \rightarrow -1} p(x)$

9) Consider the function $f(x) = \begin{cases} x^2 + kx & x \leq 5 \\ 5 \sin\left(\frac{\pi}{2}x\right) & x > 5 \end{cases}$,

In order for the function to be continuous at $x = 5$, the value of k must be

Part III: Designated Deriving! (13 points)

1) $\lim_{h \rightarrow 0} \frac{\sec(\pi + h) - \sec(\pi)}{h} =$

For #2-5, find the derivative.

2) $y = \ln(1 + e^x)$

3) $y = \csc(1 + \sqrt{x})$

4) $y = \sqrt[3]{x^3 - 4x^2}$

5) $f(x) = (x + 1)e^{3x}$

6) Consider the function $f(x) = \sqrt{x - 2}$. On what intervals are the hypotheses of the Mean Value Theorem satisfied?

7) If $xy^2 - y^3 = x^2 - 5$, then $\frac{dy}{dx} =$

8) The distance of a particle from its initial position is given by $s(t) = t - 5 + \frac{9}{(t+1)}$, where s is feet and t is minutes. Find the velocity at $t = 1$ minute in appropriate units.

Use the table below for #9-10.

X	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	2	5	$\frac{1}{2}$
3	7	-4	$\frac{3}{2}$	-1

9) The value of $\frac{d}{dx}(f \cdot g)$ at $x = 3$ is

10) The value of $\frac{d}{dx}\left(\frac{f}{g}\right)$ at $x = 1$ is

In #11-12, use the table below to find the value of the first derivative of the given functions for the given value of x .

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	0	$\frac{3}{4}$
2	7	-4	$\frac{1}{3}$	-1

11) $[f(x)]^2$ at $x = 2$ is

12) $f(g(x))$ at $x = 1$ is

Part IV: Derived and Applied! (8 points)

For #1-3, find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down.

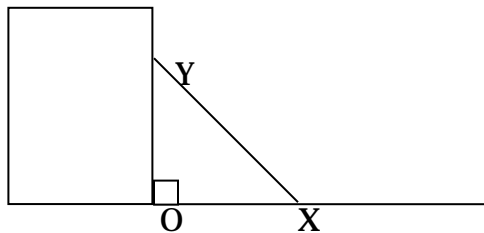
1) $f(x) = \frac{x^2 - 5}{x + 4}$

2) $y = 3x^3 - 2x^2 + 6x - 2$

3) The graph of the function $y = x^5 - x^2 + \sin x$ changes concavity at $x =$

4) For what value of x is the slope of the tangent line to $y = x^7 + \frac{3}{x}$ undefined?

5)



A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at a constant rate of $\frac{1}{2}$ foot per second.

- 2) (a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
- 2) (b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

Part V: Integral to Your Success! (12 points)

1) $\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}} dx$

2) $\int_{-\pi/6}^{\pi/6} \sec^2 x dx$

3) $\frac{d}{dx} \int_1^x \sqrt[4]{t} dt$

4) $\int \frac{x^3}{\sqrt{1+x^4}} dx$

5) $\int \frac{\csc^2 x}{\cot^3 x} dx$

6) $\int \sqrt{\tan x} \sec^2 x dx$

8) What is the average value of $y = x^3 \sqrt{x^4 + 9}$ on the interval $[0, 2]$?

9) The function f is continuous on the closed interval $[1, 9]$ and has the values given in the table. Using the subintervals $[1, 3]$, $[3, 6]$, and $[6, 9]$, what is the value of the trapezoidal approximation of $\int_1^9 f(x) dx$?

x	1	3	6	9
f(x)	15	25	40	30

10) The table below provides data points for the continuous function $y = h(x)$.

x	0	2	4	6	8	10
h(x)	9	25	30	16	25	32

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y = h(x)$ on the interval $[0, 10]$.

11) A particle moves along the x -axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is -9 , and its position is -27 .

- 1) (a) Find $v(t)$, the velocity of the particle at any time $t \geq 0$.
- 1) (b) For what values of $t \geq 0$ is the particle moving to the right?
- 1) (c) Find $x(t)$, the position of the particle at any time $t \geq 0$.

Part VI: Apply Those Integrals! (7 points)

For #1-2, find the general solution to the given differential equation.

2) 1) $\frac{dy}{dx} = y \sin x$

2) The shaded regions, R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

- 1) (a) Find the x- and y-coordinates of the three points of intersection of the graphs of f and g .
- 2) (b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g . Do not evaluate.

3) Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \leq x \leq 9$.

- 2) (a) Find the area of R .

