AP Calculus AB Summer Assignment

As Advanced Placement students, your first assignment for the beginning of the 2019-2020 school year is to come to class the very first day in top mathematical form. As Gore Vidal put it, *"The brain that doesn't feed itself eats itself."*

Calculus is a "world of change". While words like limit, continuity, derivative, and integral may seem foreign and daunting to you, I assure you that next year at this time they will seem like conversational phrases to you.

To begin our journey, you must keep yourself "mathletic" through the summer months. So, I have prepared two separate assignments to help you "burn" some time while sunbathing, cruising Europe, or playing the latest video game!

Below you will find deadlines and points assigned to each part of the summer assignment. Use this to budget your time accordingly this summer, instead of waiting until the last minute.

The due dates and point values are as follows:

- **July 15 – SECTION 1: Non-Calculator Parts I - VIII (42 points)**
- **August 5 – SECTION 2: Non-Calculator Parts IX - XIV (56 points)**
- **August 19 – SECTION 3: Calculator Section Parts XV – XVI (20 points)**

Directions:

- **IF I CANNOT MAKE OUT YOUR HANDWRITING, I AM MARKING IT WRONG!!!**
- **If the deadline is not met, you will not receive credit for that part of the summer assignment. This is NON-NEGOTIABLE.**
- **You have three ways to submit your work:**
  1. Email me your work as a WORD DOCUMENT. You can take a picture of your work and paste it into a word document. No actual pictures of your work sent via email will be accepted.
  2. Drop your work off in my mailbox at school.
  3. Mail it to the school, and it will be placed in my mailbox. Any assignments mailed to me must be postmarked NO LATER THAN THE DATE SPECIFIED TO RECEIVE CREDIT.
  If you mail, please do so under the following: *Ms. Zdanowicz (AP Calc AB Summer Work)*
  167 Broad Street
  Manasquan, NJ 08736

I honestly can say that I cannot wait to see everyone in the fall. I look forward to the wonderful mathematical insights and challenges we will all explore together. Please feel free to contact me over the summer if you have any questions with the directions. There will be no help with any of the questions. Good luck and have a safe and enjoyable summer!!

E-mail: ***kzdanowicz@manasquan.k12.nj.us***

This packet is a review of the prerequisite concepts for AP Calculus. It is to be done NEATLY and on a SEPARATE sheet of paper. All problems must show work and points will be awarded only if the correct work is shown, and that work leads to the correct answer. Have a great summer and I am looking forward to seeing you in September. ☺

Section 1: NON-CALCULATOR

[4] **Part I**. ***Simplify. Show the work that leads to your answer.***

1) $\dfrac{x-4}{x^2-3x-4}$ 2) $\dfrac{x^3-8}{x-2}$ 3) $\dfrac{5-x}{x^2-25}$ 4) $\dfrac{x^2-4x-32}{x^2-16}$

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[4] **Part II**. ***Find the equations of all vertical asymptotes***

1) $y=e^x-3$ 2) $y=\ln(x-2)$

3) $y=\dfrac{x^2+2x+4}{x^3-8}$ 4) $y=\dfrac{1}{x+2}-5$

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[2] **Part III**. ***Simplify each expression.***

1) $\dfrac{1}{x+h}-\dfrac{1}{x}$ 2) $\dfrac{2x}{x^2-6x+9}-\dfrac{1}{x+1}-\dfrac{8}{x^2-2x-3}$

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[9] **Part IV**.

Given: $f(x)=x^2+3x+1$ and $g(x)=\sqrt{x-2}$

1) (f+g)(1) 2) (g-f)(5) 3) $(f\circ g)(11)$ 4) $(g\circ f)(7)$

5) $g(g(x))$ 6) $g^{-1}(f(x))$

Use the table at right to answer #7-9.

7) $r(s(2))$ 8) $s(r^{-1}(0))$ 9) $r^{-1}(r^{-1}(s(1)))$

x	r(x)	s(x)
-3	1	2
-2	0	4
-1	2	6
0	5	-1
1	3	1
2	-1	-3
3	-3	4

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Part V. ***Miscellaneous: Follow the directions for each problem.***

1) Given $f(x) = x^2 - 2x$, find $\dfrac{f(x+h)-f(x)}{h}$. ***(2 points)***

2) Find the slope of the secant line that runs through the maximum and minimum points of the graph of $y = -5\sin 2x - 3$ when graphed on the interval $[0, \pi]$. ***(5 points)***

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Part VI. ***Factor***

1) $5x^2(x+3)^{-4} - 10x(x+3)^{-7}$

2) $14(3x-7)^{1/2} + 28(3x-7)^{-3/2}$

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Part VII. ***Simplify***

1) $e^{\ln 3}$

2) $\log_3\left(\dfrac{1}{3}\right)$

3) ln 1

4) $\ln e^7$

5) $\log_{1/2} 8$

6) ln ½

7) $e^{(1+\ln x)}$

For #8-9, expand each logarithm into a sum and or difference of logs.

8) $\log\dfrac{3\sqrt{x}}{(4+x)^2}$

9) $\ln\dfrac{\sqrt[3]{x^2-4}}{2x-1}$

For #10-11, condense each expression into a single logarithm.

10) $2\log x - 3\log y - \log(x+7)$

11) $(\ln a + \ln b) - \ln c$

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Part VIII. ***Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line…***

1) Containing the points (1,-3) and (-5,2)
2) Parallel to 2x – 3y = 7 and passes through (5,1)
3) Perpendicular to the line in problem #1, containing the point (3,4)

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Section 2: NON-CALCULATOR

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Part IX. Without a calculator, determine the exact value of each expression. Please note: You MUST be able to do this in your sleep by the time you get back to school ***WITHOUT A CALCULATOR***!!! This is one of the single most important skills necessary for success in AP Calculus!

1) $\sin 0$ 2) $\cos \frac{7\pi}{6}$ 3) $\tan \frac{\pi}{6}$ 4) $\cos(\sin^{-1} \frac{1}{2})$

5) $\sin \frac{\pi}{2}$ 6) $\cos \frac{\pi}{3}$ 7) $\tan \frac{2\pi}{3}$ 8) $\sin^{-1}(\sin \frac{7\pi}{6})$

9) $\sin \frac{3\pi}{4}$ 10) $\tan \frac{7\pi}{4}$ 11) $\tan \frac{\pi}{2}$ 12) $\sin(11\pi)$

13) $\cos\pi$ 14) $\cos\left(\frac{21\pi}{4}\right)$ 15) $\tan\left(\frac{13\pi}{3}\right)$

16) $\tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ 17) $\cos^{-1}\left(\cot\left(\frac{3\pi}{4}\right)\right)$ 18) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

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Part X

1) Graph:

(a) $y = -\frac{3}{2}\sin(2x)+1$ (b) $y = 4\cos 3(x-\pi)-1$

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Part XI. ***For each function, determine its domain. (#1-3 are 1 point each)***

1) $y = \sqrt{x-4}$ 2) $y = \sqrt{x^2-4}$ 3) $y = \sqrt{x^2+4}$

4 4) $f(x) = \frac{\ln x^2}{x+3}$ 4 5) $y = \frac{e^x}{\log(x+5)}$

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Part XII. ***Determine all points of intersection without the use of a calculator.***

1) $x^2+3x-4 = 5x+11$

2) $\cos x = \sin x$ on $\left[0, \frac{\pi}{2}\right]$

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Part XIII. ***Graph each function, without the aid of a graphing calculator. Consider each trigonometric function to be on the interval*** $[0, 2\pi]$.

1) y = sin x

2) y = cos x

3) y = tan x

4) $y = (x-3)^2 + 2$

5) $y = \frac{1}{x+1} - 1$

6) $y = \frac{x^2 - 4}{x+2}$

7) y = e^x

8) y = $\sqrt{x}$

9) y = $\sqrt[3]{x}$

10) y = ln x

11) y = |x+3| - 2

12) y = $\frac{1}{x}$

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13) $y = \begin{cases} x^2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \le x \le 3 \\ 4 & \text{if } x > 3 \end{cases}$

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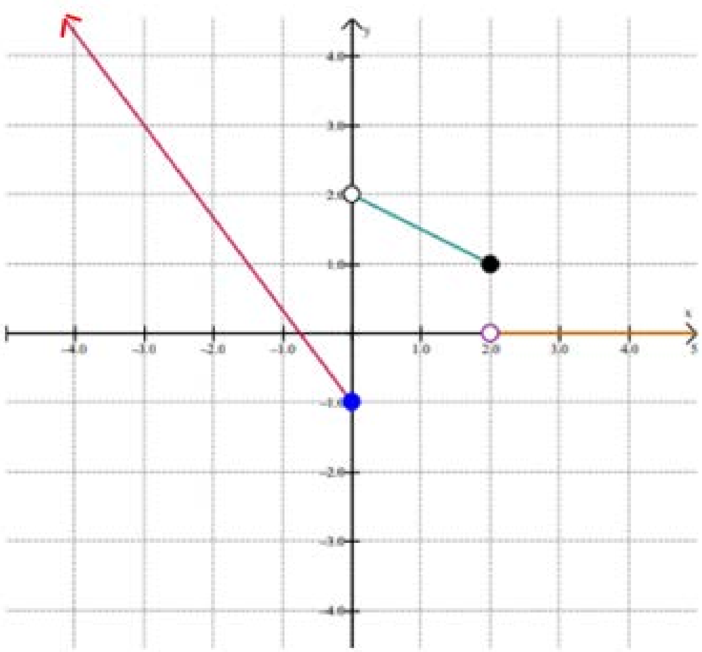
14) Write the equation for the piecewise function shown below:



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Part XIV. ***Is this function continuous? If no, explain why it is discontinuous.***

1) $g(x) = \begin{cases} 2x - 4, & x < 3 \\ -x + 5, & x \ge 3 \end{cases}$

2) $b(x) = \frac{x(3x+1)}{3x^2 - 5x - 2}$

3) $h(x) = \frac{\sqrt{x^2 - 10x + 25}}{x - 5}$

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Section 3: GRAPHING CALCULATOR

This part is designed to help you become comfortable with your graphing calculator. You may need to read the manual to understand how your calculator works in some situations, if you do not know already. It is important that you gain these skills ASAP so that we can spend time talking about calculus rather than how to use the calculator. Note: Logarithmic and Exponential Functions do not have endpoints (even if it "looks" like it does on your calculator).

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Part XV.

1) Given: $f(x) = x^4 - 3x^3 + 2x^2 - 7x - 11$
   Find all roots to the nearest 0.001.

2) Given: $f(x) = 3\sin 2x - 4x + 1$ from $[-2\pi, 2\pi]$
   Find all roots to the nearest 0.001.
   Note: All trig functions are done in radian mode.

3) Given: $f(x) = 100x^3 - 203x^2 + 103x - 1$
   Find all roots to the nearest 0.001.

4) Given: $f(x) = |x-3| + |x| - 6$
   Find all roots to the nearest 0.001.

5) Given: $f(x) = |x| - |x-6|$
   Find all roots to the nearest 0.001.

Solve for the inequality.

6) $x^2 - 2x - 5 \geq 0$

7) $x^3 - 4x < 0$

8) Given $f(x) = x^2 - 5x + 2$ and $g(x) = 3 - 2x$; Find the coordinates of any points of intersection.

9) How many times does the graph of $y = 0.1x$ intersect the graph of $y = \sin(2x)$?

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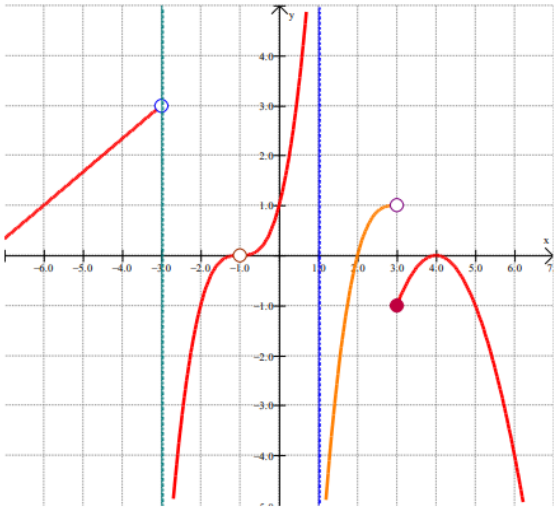
Part XVI. ***LIMITS - A limit tells us the value that a function approaches as that function's inputs get closer and closer to some number. The idea of a limit is the basis of all calculus.***

This next part you need to know how to do before class begins…LIMITS! Most of you know how to do this but some may not. Please go onto https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-2/v/introduction-to-limits-hd . Watch the limits intro video, limits intro examples, and do the practice. Then answer the questions below.

Using the graph at the right, answer the following.



1) $\lim_{x\to -6} f(x) =$

2) $\lim_{x\to -1} f(x) =$

3) $\lim_{x\to 0} f(x) =$

4) $\lim_{x\to 3} f(x) =$

5) f (3) =

Now watch these videos to understand the algebraic part of finding a limit.
https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-5b/v/limit-by-substitution
https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-6/v/limit-example-1
https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-6/v/limits-by-rationalizing
Then answer the questions below.

Find the limits.

6) $\lim_{x\to 1} \frac{x^2-2x-5}{x+1}$

7) $\lim_{x\to 5} \frac{x-5}{x+2}$

8) $\lim_{x\to 5} \frac{x+2}{x-5}$

9) $\lim_{x\to 4} \frac{2x^3-7x^2-4x}{x-4}$

10) $\lim_{x\to 9} \frac{\sqrt{x}-3}{9-x}$

11) $\lim_{x\to -2} \frac{x^3+8}{x+2}$

SECTION 4: *YOU DO NOT SUBMIT THESE!* *YOU MUST KNOW WHAT THESE GRAPHS LOOK LIKE AND HOW TO GRAPH IN YOUR CALCULATOR.*

1) Parent Function: $y = x^2$

a) $y = x^2 - 5$
b) $y = x^2 + 3$
c) $y = (x-10)^2$
d) $y = (x+8)^2$
e) $y = 4x^2$
f) $y = 0.25x^2$
g) $y = -x^2$
h) $y = -(x+3)^2 + 6$
I) $y = (x+4)^2 - 8$
j) $y = -2(x+1)^2 + 4$
k) $y = -(x-6)^2 - 6$
l) $y = -3(x+2)^2 - 2$

2) Parent Function: $y = \sin(x)$ (set mode to RADIANS)

a) $y = \sin(2x)$
b) $y = \sin(x) - 2$
c) $y = 2\sin(x)$
d) $y = 2\sin(2x) + 2$

3) Parent Function: $y = \cos(x)$

a) $y = \cos(3x)$
b) $y = \cos\left(\frac{x}{2}\right)$
c) $y = 2\cos(x) + 2$
d) $y = -2\cos(x) - 1$

4) Parent Function: $y = x^3$

a) $y = x^3 + 2$
b) $y = -x^3$
c) $y = x^3 - 5$
d) $y = -x^3 + 3$
e) $y = (x - 4)^3$
f) $y = (x - 1)^3 - 4$
g) $y = -2(x + 2)^3 + 1$
h) $y = x^3 + x$

5) Parent Function: $y = \sqrt{x}$

a) $y = \sqrt{x} - 2$
b) $y = \sqrt{-x}$
c) $y = \sqrt{x} + 5$
d) $y = \sqrt{6 - x}$
e) $y = -\sqrt{x}$
f) $y = -\sqrt{-x}$
g) $y = \sqrt{x + 2}$
h) $y = \sqrt{2x - 6}$
i) $y = -2\sqrt{x}$
j) $y = -\sqrt{4 - x}$

6) Parent Function: $y = \ln(x)$

a) $y = \ln(x+3)$
b) $y = \ln(x) + 3$
c) $y = \ln(x - 2)$
d) $y = \ln(-x)$
e) $y = -\ln(x)$
f) $y = \ln(|x|)$
g) $y = \ln(2x) - 4$
h) $y = -3\ln(x) + 1$

7) Parent Function: $y = e^x$

a) $y = e^{2x}$
b) $y = e^{x-2}$
c) $y = e^{2-x}$
d) $y = e^{2x} + 3$
e) $y = -e^x$
f) $y = e^{-x}$
g) $y = 2 - e^x$
h) $y = e^{0.5x}$

8) Parent Function: $y = a^x$

a) $y = 5^x$
b) $y = 2^x$
c) $y = 3^{-x}$
d) $y = -2^x$
e) $y = 4^{x-3}$
f) $y = 2^{x-3} + 2$

9) Parent Function: $y = \frac{1}{x}$

a) $y = \frac{1}{x-2}$
b) $y = -\frac{1}{x}$
c) $y = \frac{1}{x+4}$
d) $y = \frac{2}{5-x}$