AP Calculus AB Summer Assignment

As Advanced Placement students, your first assignment for the beginning of the 2020-2021 school year is to come to class the very first day in top mathematical form. As Gore Vidal put it, *"The brain that doesn't feed itself eats itself."*

Calculus is a "world of change". While words like limit, continuity, derivative, and integral may seem foreign and intimidating to you, I assure you that next year at this time they will seem like conversational phrases to you!

To begin our journey, you must keep yourself "mathletic" through the summer months. So, I have prepared a three section (non-calculator, calculator, limits) assignment to help you "burn" some time during the summer. Please budget your time accordingly this summer, instead of waiting until the last minute.

I honestly can say that I cannot wait to see everyone in the fall. I look forward to the wonderful mathematical insights and challenges we will all explore together. Please feel free to contact me over the summer if you have any questions with the directions. There will be no help with any of the questions. Good luck and have a safe and enjoyable summer!!

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Due Date: First class period (week of September 7th)
You will hand this in at the beginning of the period on the first class of the year…with a big smile. ☺

**If for some reason we are not back in school, you will email me your summer assignment by September 8th.

Email Directions: Email me your work as a WORD DOCUMENT. You can take a picture of your work and paste it into a word document. No actual pictures of your work sent via email will be accepted.

This packet is a review of the prerequisite concepts for AP Calculus. It is to be done NEATLY and on a SEPARATE sheet of paper. If I cannot make out your handwriting, I am marking it wrong. All problems must show work and points will be awarded only if the correct work is shown, and that work leads to the correct answer.

Section 1: NON-CALCULATOR

Part I: Simplify. Show the work that leads to your answer.

1) $\frac{x-4}{x^2-3x-4}$ 2) $\frac{x^3-8}{x-2}$ 3) $\frac{5-x}{x^2-25}$ 4) $\frac{x^2-4x-32}{x^2-16}$

Part II: Simplify each expression.

1) $\frac{1}{x+h}-\frac{1}{x}$ 2) $\frac{2x}{x^2-6x+9}-\frac{1}{x+1}-\frac{8}{x^2-2x-3}$

Part III: Follow each of the directions.

Given: $f(x)=x^2+3x+1$ and $g(x)=\sqrt{x-2}$

1) (f+g)(1) 2) (g-f)(5) 3) $(f\circ g)(11)$ 4) $(g\circ f)(7)$

5) $g(g(x))$ 6) $g^{-1}(f(x))$

Use the table at right to answer #7-9.

x	r(x)	s(x)
-3	1	2
-2	0	4
-1	2	6
0	5	-1
1	3	1
2	-1	-3
3	-3	4

7) $r(s(2))$ 8) $s(r^{-1}(0))$ 9) $r^{-1}(r^{-1}(s(1)))$

10) Given $f(x)=x^2-2x$, find $\frac{f(x+h)-f(x)}{h}$.

Part IV: Factor out the GCF, and then simplify.

1) $5x^2(2x-3)^5-8y(2x-3)^6$

2) $3x^2(x+3)^{\frac{1}{2}}+8x(x+3)^{\frac{3}{2}}$

3) $5x^2(x+3)^{-4}-10x(x+3)^{-7}$

4) $6x(x-5)^{\frac{1}{2}}+8x(x-5)^{-\frac{1}{2}}$

Part V: Simplify.

1) $e^{\ln 3}$

2) $\log_3\left(\frac{1}{3}\right)$

3) $\ln 1$

4) $\ln e^7$

5) $\log_{1/2} 8$

6) $\ln \frac{1}{2}$

7) $e^{(1+\ln x)}$ ***Expand #7 by properties of exponents.***

For #8-9, expand each logarithm into a sum and or difference of logs.

8) $\log\frac{3\sqrt{x}}{(4+x)^2}$

9) $\ln\frac{\sqrt[3]{x^2-4}}{2x-1}$

For #10-11, condense each expression into a single logarithm.

10) $2\log x-3\log y-\log(x+7)$

11) $(\ln a+\ln b)-\ln c$

Part VI: Write a linear equation by using the point-slope form: $y-y_1=m(x-x_1)$

1) Containing the points (1,-3) and (-5,2)
2) Parallel to 2x – 3y = 7 and passes through (5,1)
3) Perpendicular to the line in problem #1, containing the point (3,4)

Part VII: Find the domain of each function in interval form.

1) $f(x)=\sqrt{x-2}$

2) $f(x)=\sqrt{x^2-4}$

3) $f(x)=\sqrt[3]{2x+3}$

4) $f(x)=\frac{2}{\sqrt{x}}$

5) $f(x)=\frac{4}{x+3}$

6) $f(x)=\frac{x+4}{x-1}$

Part VIII: Unit Circle…This is one of the single most important skills for success in this class! You must know the unit circle back and forth – you will not have a unit circle on the AP Exam so know it! Find the exact value for each problem.

1) $\sin 0$ 2) $\cos \frac{7\pi}{6}$ 3) $\tan \frac{\pi}{6}$ 4) $\cos(\sin^{-1} ½)$

5) $\sin \frac{\pi}{2}$ 6) $\cos \frac{\pi}{3}$ 7) $\tan \frac{2\pi}{3}$ 8) $\sin^{-1}(\sin \frac{7\pi}{6})$

9) $\sin \frac{3\pi}{4}$ 10) $\tan \frac{7\pi}{4}$ 11) $\tan \frac{\pi}{2}$ 12) $\sin(11\pi)$

13) $\cos \pi$ 14) $\cos\left(\frac{21\pi}{4}\right)$ 15) $\tan\left(\frac{13\pi}{3}\right)$

16) $\tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ 17) $\cos^{-1}\left(\cot\left(\frac{3\pi}{4}\right)\right)$ 18) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

Part IX: Functions and their graphs.

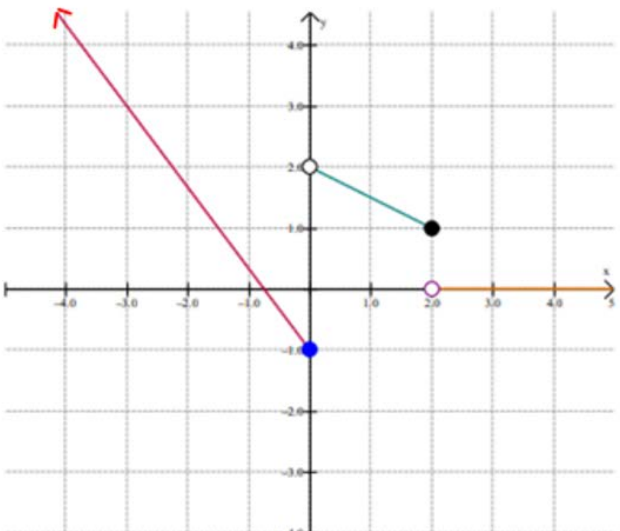
For # 1 – 4, if the parent graph is $y = \sqrt{x}$, describe how the graph transforms to:

1) $y = 2\sqrt{x} - 5$ 2) $y = -\sqrt{x+2} + 6$ 3) $-y = -\frac{1}{3}\sqrt{4-x}$ 4) $y = \sqrt{2x-6}$

5) Graph one cycle: $y = -\frac{3}{2}\sin(2x) + 1$ 6) Graph one cycle: $y = 4\cos 3(x-\pi) - 1$

7) Graph the piecewise function. $y = \begin{cases} x^2 & \text{if } x < 0 \\ x+2 & \text{if } 0 \le x \le 3 \\ 4 & \text{if } x > 3 \end{cases}$

8) Write the equation for the piecewise function shown below:



Part X: Is this function continuous? If no, explain why it is discontinuous.

1) $g(x)=\begin{cases}2x-4, & x<3\\ -x+5, & x\geq 3\end{cases}$

2) $b(x)=\dfrac{x(3x+1)}{3x^2-5x-2}$

3) $h(x)=\dfrac{\sqrt{x^2-10x+25}}{x-5}$

Part XI: Determine all points of intersection without the use of a calculator.

1) $x^2+3x-4=5x+11$

2) $\cos x=\sin x$ on $\left[0,\dfrac{\pi}{2}\right]$

Section 2: GRAPHING CALCULATOR

This part is designed to help you become comfortable with your graphing calculator. You may need to read the manual to understand how your calculator works in some situations, if you do not know already. It is important that you gain these skills ASAP so that we can spend time talking about calculus rather than how to use the calculator. Note: Logarithmic and Exponential Functions do not have endpoints (even if it "looks" like it does on your calculator).

Part XII: Graph each function on you calculator (if necessary) and then draw a sketch. These are graphs that you should know without a calculator by the time class starts. Trig function – graph on interval $[0,2\pi]$ ***and you calculator should be in radians.***

1) y = sin x

2) y = cos x

3) y = tan x

4) $y=x^2$

5) $y=x^3$

6) $y=|x|$

7) $y=\dfrac{1}{x}$

8) $y=\sqrt{x}$

9) $y=\sqrt[3]{x}$

10) $y=e^x$

11) y = ln x

12) $y=2^x$

Part XIII: Find the domain in interval form.

1) $f(x)=\dfrac{\ln x^2}{x+3}$

2) $y=\dfrac{e^x}{\log(x+5)}$

Part XIV: Graphing functions.

For # 1 – 3, find the roots/zeros to the nearest thousandth.

1) $f(x) = x^4 - 3x^3 + 2x^2 - 7x - 11$

2) $f(x) = 3\sin(2x) - 4x + 1 \quad from \ [-2\pi, 2\pi]$

3) $f(x) = |x - 3| + |x| - 6$

4) Given $f(x) = x^2 - 5x + 2$ and $g(x) = 3 - 2x$; Find the coordinates of any points of intersection.

5) How many times does the graph of y = 0.1 intersect the graph of y = sin(2x) from [0, 2π].

Part XV: Find the VA, Hole and HA (if they exist), and sketch showing the graph (solid) and asymptotes (dotted).

1) $y = e^x - 3$

2) $y = \ln(x - 2)$

3) $y = \dfrac{x^2 - 4}{x + 2}$

4) $y = \dfrac{1}{x + 2} - 5$

5) $y = \dfrac{x^2 + 2x + 4}{x^3 - 8}$

Part XVI: Solve for the inequality.

1) $x^2 - 2x - 5 \geq 0$

2) $x^3 - 4x < 0$

Section 3: LIMITS

A limit is the y-value that a function approaches but may not necessarily attain. The idea of a limit is the basis of all calculus. You need to know how to find a limit before our class begin.

Part XVII: Please go onto https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-2/v/introduction-to-limits-hd . Watch the limits intro video, limits intro examples, and do the practice.

For #1 – 6, find the limit or value using the graph provided.

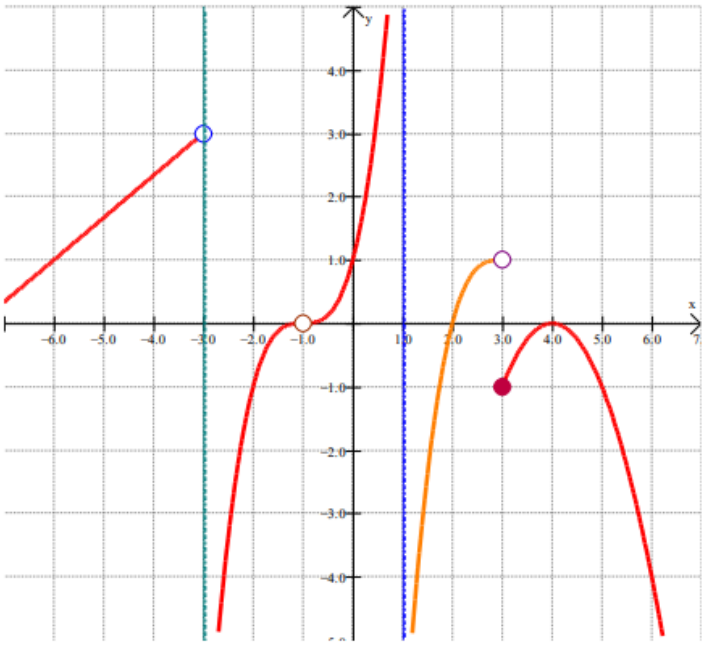
1) $\lim_{x\to -6} f(x) =$

2) $\lim_{x\to -1} f(x) =$

3) $\lim_{x\to 0} f(x) =$

4) $\lim_{x\to 3} f(x) =$

5) f (3) =



Part XVIII: Now watch these videos to understand the algebraic part of finding a limit.
https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-5b/v/limit-by-substitution
https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-6/v/limit-example-1
https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-6/v/limits-by-rationalizing

For # 1 – 6, find the limit. Show all work!

1) $\lim_{x\to 1} \dfrac{x^2-2x-5}{x+1}$

2) $\lim_{x\to 5} \dfrac{x-5}{x+2}$

3) $\lim_{x\to 5} \dfrac{x+2}{x-5}$

4) $\lim_{x\to 4} \dfrac{2x^3-7x^2-4x}{x-4}$

5) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{9-x}$

6) $\lim_{x\to -2} \dfrac{x^3+8}{x+2}$