3.4 Linear Programming

Using your phone, find a suitable definition of the term "linear programming":

Linear programming is an extension of solving linear inequalities.

_____ is a technique that identifies the minimum or maximum value of some quantity. Linear programming is widely used in the management of business to find optimal solutions to complex problems.

The quantity is modeled with an ______.

Objective function:

Limits on the variables in the objective function are _____, written as linear inequalities. The constraints form a system of inequalities. Real-world constraint:

The graph of the system of constraints is called a ______. (Note: Feasible means "doable" or "suitable".) The feasible region contains all the points that satisfy all the constraints. Constraints are sometimes called restrictions.

If an objective function has a maximum or a minimum value, then it must occur at one or more vertices of the feasible region. Moreover, the objective function will have both a maximum and minimum value if the feasible region is bounded.

You can use linear programming to solve many real-world problems. On this level, vertices of the feasible region will be whole number coordinates and make sense in terms of the situation. Otherwise, a more detailed analysis would be necessary.

Graph the feasible region for each set of constraints. Identify the vertices of the feasible region.



Find the maximum and minimum values, if they exist, of each objective function.





Example 7: Use the constraints in the previous example with the objective function P = 2x + 3y. Find the values of x and y that maximize and minimize P. Find the value of P at each point.

<u>Example 8</u>: A small company produces knitted afghans and sweaters and sells them through a chain of specialty stores. The company is to supply the stores with a total of no more than 100 afghans and sweaters per day. The stores guarantee that they will sell at least 10 and no more than 60 afghans per day and at least 20 sweaters per day.

The company makes a profit of \$10 on each afghan and a profit of \$12 on each sweater.

Let x represent:

y:

- a. Write a system of inequalities to represent the constraints.
- b. Graph the feasible region.



- c. Write an objective function for the company's total profit, P, from the sales of afghans and sweaters.
- d. How can the company maximize its profit?

e. How much is the maximum profit?

<u>Example 9</u>: A furniture manufacturer can make from 30 to 60 tables a day and from 40 to 100 chairs a day. The profit on a table is \$150, and the profit on a chair is \$65. How many tables and chairs should they make per day to maximize profit? How much is the maximum profit?



<u>Example 10</u>: Sal's Landscaping Company has crews who mow lawns and prune shrubbery. The company schedules 1 hour for mowing jobs and 3 hours for pruning jobs. Each crew's schedule is set up for a maximum of 9 hours per day. Each crew is scheduled for no more than 2 pruning jobs per day. On the average, the charge for mowing a lawn is \$40 and the charge for pruning shrubbery is \$120. Find a combination of mowing lawns and pruning shrubs that will maximize the income the company receives per day for one of its crews.



Wrapping it Up!

In a linear programming problem, what values do you test to find the values that maximize or minimize the objective function?