2023 AP Calculus (BC) Summer Assignment (60 points)

This packet is a review of some Precalculus topics and some Calculus topics. It is to be done NEATLY and on a SEPARATE sheet of paper. Use your discretion as to whether you should use a calculator or not. When in doubt, think about whether you would have used the GC in Honors Precalc or Calc AB – that should guide you! Points will be awarded only if the correct work is shown, and that work leads to the correct answer. Have a great summer and I am looking forward to seeing you in September. ☺

Parts 1 – 3: Due______Wednesday, July 26, 2023________

Parts 4 – 6: Due______Wednesday, August 30, 2023______

All work needs to be received by the above due date. Can be submitted on Canvas in pdf form or dropped off in Main Office (mailbox – Canonaco)

LATE work will not be accepted!

Part I: First, let's whet your appetite with a little Precalc! (10 points)

2 1) For what value of k are the two lines $2x + ky = 3$ and $x + y = 1$

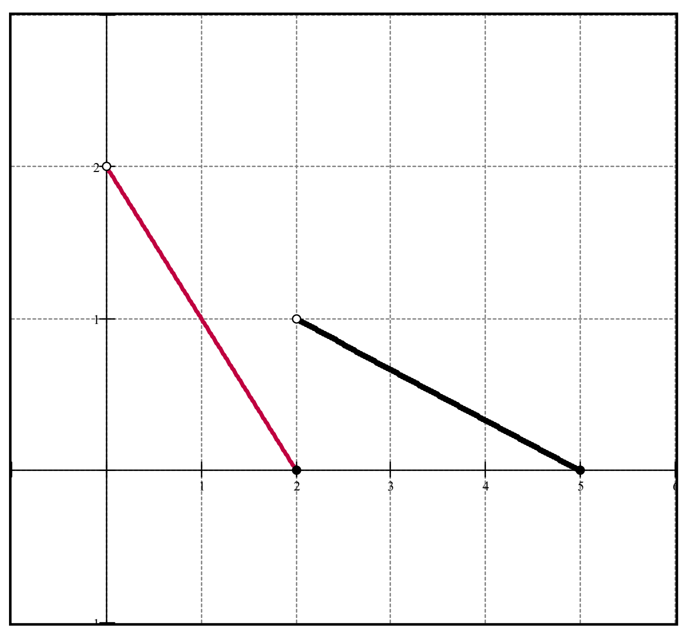
(a) parallel? (b) perpendicular?

2 2) Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4) in slope intercept form.

2 3) Graph the function shown below. Also indicate any key points and state the domain and range.

$$f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \dfrac{3}{2}x + \dfrac{3}{2}, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$$

2 4) Write a piecewise formula for the function shown. Include the domain of each piece!



2 5) Graph the function $y = 3e^{-x} - 2$ and indicate asymptote(s). State its domain, range, and intercepts.

Part II: Unlimited and Continuous! (10 points)

For #1-2 below, find the limits, if they exist.(#1-8 are 1 pt each)

1) $\lim_{x\to 4} \frac{2x^3 - 7x^2 - 4x}{x-4}$

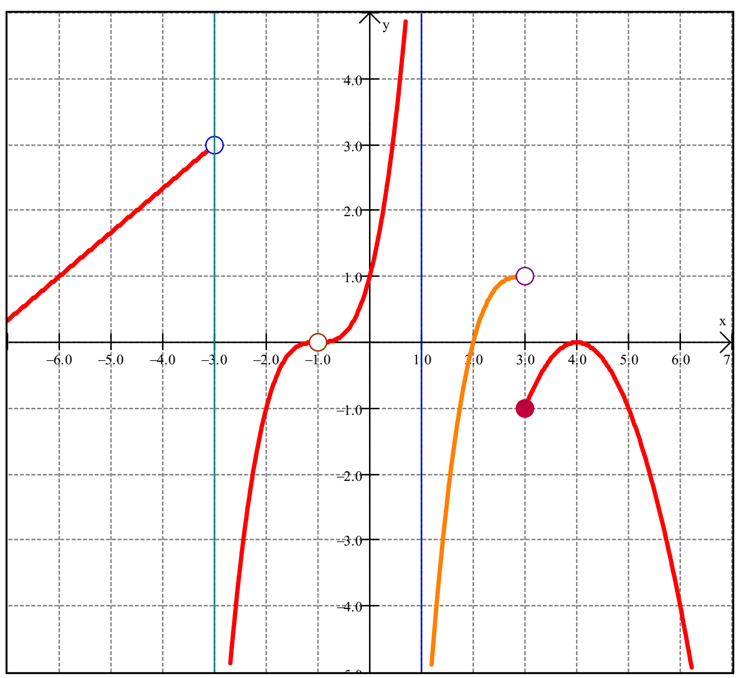
2) $\lim_{x\to 9} \frac{\sqrt{x} - 3}{9-x}$

For #3-4, explain why each function is discontinuous and determine if the discontinuity is removable or non-removable.

3) $g(x) = \begin{cases} 2x-3, & x < 3 \\ -x+5, & x \geq 3 \end{cases}$

4) $h(x) = \frac{\sqrt{x^2 - 10x + 25}}{x-5}$

For #5-8, determine if the following limits exist, based on the graph below of p(x). If the limits exist, state their value. Note that x = -3 and x = 1 are vertical asymptotes.



5) $\lim_{x\to 1^-} p(x)$

6) $\lim_{x\to -3^-} p(x)$

7) $\lim_{x\to 3} p(x)$

8) $\lim_{x\to -1} p(x)$

2 9) Consider the function $f(x) = \begin{cases} x^2 + kx & x \leq 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & x > 5 \end{cases}$,

In order for the function to be continuous at $x = 5$, the value of k must be

Part III: Designated Deriving! (12 points)

1 1) $\lim_{h \to 0} \dfrac{\sec(\pi + h) - \sec(\pi)}{h} =$

For #2-5, find the derivative.

1 2) $y = \ln(1 + e^x)$

1 3) $y = \csc(1 + \sqrt{x})$

1 4) $y = \sqrt[7]{x^3 - 4x^2}$

1 5) $f(x) = (x+1)e^{3x}$

1 6) Consider the function $f(x) = \sqrt{x-2}$. On what intervals are the hypotheses of the Mean Value Theorem satisfied?

1 7) If $xy^2 - y^3 = x^2 - 5$, then $\dfrac{dy}{dx} =$

1 8) The distance of a particle from its initial position is given by $s(t) = t - 5 + \dfrac{9}{(t+1)}$, where *s* is feet and *t* is minutes. Find the velocity at *t* = 1 minute in appropriate units.

Use the table below for #9-10.

x	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	4	2	5	½
3	7	-4	$\frac{3}{2}$	-1

1 9) The value of $\dfrac{d}{dx}(f \cdot g)$ at x = 3 is

1 10) The value of $\dfrac{d}{dx}\left(\dfrac{f}{g}\right)$ at x = 1 is

In #11-12, use the table below to find the value of the first derivative of the given functions for the given value of x.

x	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	3	2	0	$\frac{3}{4}$
2	7	-4	$\frac{1}{3}$	-1

1 11) $\dfrac{d}{dx}[f(x)]^2$ at x = 2 is

1 12) $\dfrac{d}{dx} f(g(x))$ at x = 1 is

Part IV: Derived and Applied! (8 points)

For #1-3, find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down.

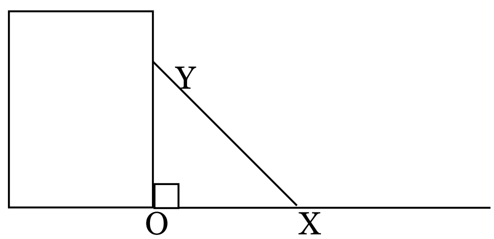
1 1) $f(x) = \dfrac{x^2 - 5}{x + 4}$

1 2) $y = 3x^3 - 2x^2 + 6x - 2$

1 3) The graph of the function $y = x^5 - x^2 + \sin x$ changes concavity at $x =$

1 4) For what value of x is the slope of the tangent line to $y = x^7 + \dfrac{3}{x}$ undefined?

5)



A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at a constant rate of ½ foot per second.

2 (a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.

2 (b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

Part V: Integral to Your Success! (12 points)

1 1) $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}} dx$

1 2) $\int_{-\pi/6}^{\pi/6} \sec^2 x dx$

1 3) $\frac{d}{dx} \int_{1}^{x} \sqrt[4]{t}\, dt$

1 4) $\int \frac{x^3}{\sqrt{1+x^4}} dx$

1 5) $\int \frac{\csc^2 x}{\cot^3 x} dx$

1 6) $\int \sqrt{\tan x} \sec^2 x dx$

1 8) What is the average value of $y = x^3\sqrt{x^4+9}$ on the interval [0, 2]?

1 9) The function f is continuous on the closed interval [1, 9] and has the values given in the table. Using the subintervals [1, 3], [3, 6], and [6, 9], what is the value of the trapezoidal approximation of $\int_{1}^{9} f(x)dx$?

x	1	3	6	9
$f(x)$	15	25	40	30

1 10) The table below provides data points for the continuous function $y = h(x)$.

x	0	2	4	6	8	10
$h(x)$	9	25	30	16	25	32

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y = h(x)$ on the interval [0, 10].

11) A particle moves along the x-axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is -9, and its position is -27.

1 (a) Find $v(t)$, the velocity of the particle at any time $t \geq 0$.
1 (b) For what values of $t \geq 0$ is the particle moving to the right?
1 (c) Find $x(t)$, the position of the particle at any time $t \geq 0$.

Part VI: Apply Those Integrals! (8 points)

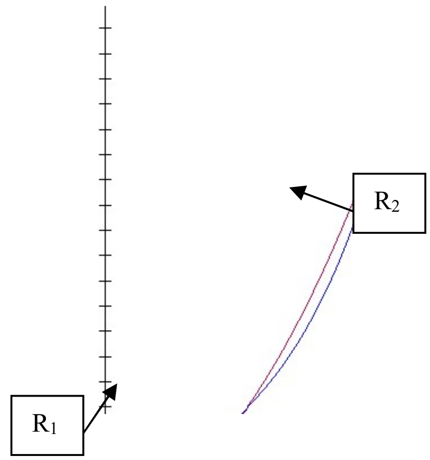
For #1-2, find the general solution to the given differential equation.

2 1) $\dfrac{dy}{dx} = y \sin x$

2) The shaded regions, R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

1 (a) Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.

2 (b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.



3) Let R be the region in the first quadrant under the graph of $y = \dfrac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

1 (a) Find the area of R.

2 (b) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.