Rolle's Theorem
The Mean Value Theorem
Homework

For #1-5, determine whether Rolle's Theorem can be applied to f on the indicated interval. If Rolle's Theorem can be applied, find all values of c in the interval such that $f'(c) = 0$.

1) $f(x) = x^2 - 3x + 2 \quad [1,2]$

2) $f(x) = (x-1)(x-2)(x-3) \quad [1,3]$

3) $f(x) = x^{2/3} - 1 \quad [-8,8]$

4) $f(x) = \sin 2x \quad \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

5) $f(x) = \frac{6x}{\pi} - 4\sin^2 x \quad \left[0, \frac{\pi}{6}\right]$

6) The height of a ball t seconds after it is thrown upward from a height of 32 feet and with an initial velocity of 48 feet per second is $f(t) = -16t^2 + 48t + 32$.
   (a) Verify that $f(1) = f(2)$
   (b) According to Rolle's Theorem, what must be the velocity at some time in the interval [1, 2]?

For #7-11, apply the Mean Value Theorem to f on the indicated interval. In each case, find all values of c in the interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

7) $f(x) = x^{2/3} \quad [0,1]$

8) $f(x) = \frac{x+1}{x} \quad \left[\frac{1}{2}, 2\right]$

9) $f(x) = \sqrt{x-2} \quad [2,6]$

10) $f(x) = x^3 \quad [0,1]$

11) $f(x) = \sin x \quad [0, \pi]$

In #12 and #13, explain why the MVT does not apply to the function on the interval [0,6].

12) $f(x) = \frac{1}{x-3}$

13) $f(x) = |x-3|$

14) The height of an object t seconds after it is dropped from a height of 500 meters is $s(t) = -4.9t^2 + 500$.

(a) Find the average velocity of the object during the first 3 seconds.
(b) Use the MVT to verify that at some time during the first 3 seconds of fall the instantaneous velocity equals the average velocity. Find that time.

15) A company introduces a new product for which the number of units sold S is $S(t) = 200\left(5 - \frac{9}{2+t}\right)$ where t is the time in months.

(a) Find the average value of S(t) during the first year.
(b) During what month does $S'(t)$ equal the average value during the first year.

For #15, state why Rolle's Theorem does not apply to the function even though there exist a and b such that $f(a) = f(b)$.

15)